Surface Mass Injection at Supersonic and Hypersonic Speeds as a Problem in Turbulent Mixing. Part II: Axially-Symmetric Flow

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The effects of gas injection normal to the surface of two-dimensional bodies in supersonic or hypersonic flight were determined in the previously published Part I of the present study. In Part II the effects of axial-symmetry are considered, and for the two-dimensional case, it is found that significant streamline inclinations and surface pressures are generated by the action of turbulent viscous dissipation alone. For a "cold" surface at hypersonic speeds, similarity parameters are obtained which relate the induced streamline inclinations and surface pressures to the nondimensional blowing rate. The exact form of these relationships for a cone depends on the order of magnitude of the ratio of induced streamline deflection angle to cone angle. When the surface is not cold relative to the freestream total temperature, the magnitude of the induced streamline deflection depends on the velocity profile shape factor. For the two-dimensional case this shape factor becomes infinite as the nondimensional blowing rate approaches a fixed value corresponding to boundary-layer "blowoff." For the axially symmetric case, solutions can be obtained for blowing rates much larger than the rate corresponding to the two-dimensional blowoff value.

Nomenclature

B' = velocity ratio at surface, $-\frac{1}{2}(\overline{c_f}/\overline{c_{f_0}})\overline{c_{f_0}}/\lambda_e$
c_f = local skin friction coefficient, $\tau_w/\frac{1}{2}\rho_e u_e^2$
C_p = pressure coefficient, $(p - p_\infty)/\frac{1}{2}\rho_\infty u_\infty^2$
$f(\eta)$ = nondimensional stream function $[f_0 = -f(0)]$
F = $(T_w/T_e + z)H_i + [(\gamma - 1)/2]M_e^2$
$G(Y/\theta_i)$ = normalized turbulent eddy diffusivity, $\epsilon/\tilde{\epsilon}_0$
$g(x)$ = r_0/L
h = static enthalpy
h_s = total enthalpy
H_1 = velocity profile shape factor, $\delta_1{}^*/\theta_1$
H_a = $\delta_a{}^*/\theta_a$
H_i = $\delta_i{}^*/\theta_i$
K = hypersonic similarity parameter, $M_\infty(\Theta + \theta_c)$
K_c = hypersonic similarity parameter, $M_\infty\theta_c$
K_I = hypersonic similarity parameter, $M_\infty\Theta(\delta)$
K_s = hypersonic similarity parameter, $M_\infty\theta_s$
K_w = hypersonic similarity parameter, $M_\infty\Theta_w$
K_θ = reciprocal of wake-like turbulent Reynolds number $K_\theta = 0.06$
L = characteristic length
$\dot{m},\dot{m}_i$ = mass flux; surface mass injection rate
M = Mach number, u/a
r = radius to generic point
r_0 = local body radius
r_e = local viscous layer radius, $r_0 + \delta\cos\alpha$
p = static pressure
R_e = Reynolds number, $\rho_e u_e L/\mu_e$
S = $h_s/h_{s_e} - 1$
T = static temperature
T_s = total temperature
u,v = velocity components parallel and normal to surface, respectively
U,V = transformed velocity
x,y = coordinates along and normal to surface, respectively
$\bar{y}$ = $y[1 + (y/2r_0)\cos\alpha]$
y = $\rho_e a_e r_0/p_2 a_2 L\bar{y}$
$\hat{y}$ = $\tilde{y}/\xi$
X,Y = transformed coordinates, Eq. (26)
z = $\frac{1}{\delta_i{}^*}\int_0^{\delta_i}\frac{U}{U_e}dY$
α = local body slope
β_i = pressure gradient parameter, $\beta_i/(1 - \beta_i) = X/M_e\,(dM_e/dX)$
γ = ratio of specific heats
δ = viscous layer thickness
δ^* = displacement thickness:
$$\int_0^{\delta^*}\rho_e u_e r dy = \int_0^{\delta}(\rho_e u_e - \rho u)r dy$$
$\delta_a{}^*$ = modified displacement thickness
$$\int_0^{\delta}\left(1 - \frac{\rho u r}{\rho_e u_e r_e}\right)dy$$
δ_1 = $\delta^*[1 - (\delta^*/2r_0)\cos\alpha]$
$\bar{\delta}$ = $\delta[1 + (\delta/2r_0)\cos\alpha]$
ϵ = turbulent eddy diffusivity, $= \rho\epsilon\partial u/\partial y$
$\tilde{\epsilon}_0$ = wake-like diffusivity, $K_\theta U_e\theta_i$
η = nondimensional distance normal to surface, $Y/\xi(X)$
$\eta_{\delta_i^*}$ = $\frac{\delta_i{}^*}{\xi(X)} = \int_0^\infty(1 - f')d\eta$
η_{θ_i} = $\frac{\theta_i}{\xi(X)}\int_0^\infty f'(1 - f')d\eta$
θ = momentum thickness:
$$\int_0^{\theta}\rho_e u_e{}^2 r dy = \int_0^{\delta}\rho u(u_e - u)r dy$$
θ_a = modified momentum thickness:
$$\int_0^{\delta}\frac{\rho u r}{\rho_e u_e r_e}\left(1 - \frac{u}{u_e}\right)dy$$
θ_c = cone half-angle
θ_s = shock wave angle

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$\Theta(\delta)$ = induced streamline inclination with respect to local tangent to surface
Θ_w = wedge half-angle
$\lambda_e;\lambda_\infty$ = $\dot{m}_i/\rho_e u_e$; $\dot{m}_i/\rho_\infty u_\infty$
θ_1 = $\theta[1 + (\theta/2r_0)\cos\alpha]$
Λ_∞ = hypersonic blowing parameter, $[(\gamma - 1)/2]M_\infty{}^3\lambda_\infty$
$\xi(X)$ = distance parameter along surface
ρ = density
$\tilde{\sigma}$ = transverse curvature parameter $\rho_e a_e r_0{}^2/2\rho_2 a_2 L \cos\alpha$
$\hat{\sigma}$ = σ/ξ
τ = shear stress
ψ = stream function

Subscripts

e = local "external" inviscid flow
w = surface
∞ = freestream
i = equivalent low-speed two-dimensional flow (except $\dot{m}_i$)
2 = local external inviscid flow just behind leading edge shock
0 = zero mass injection (except f_0 and r_0)

Superscript‡

— = average value

1. Introduction

IN Part I (seeRef.1) of our study of the effects of surface mass injection at supersonic and hypersonic speeds, it was found that an analysis based on the turbulent mixing of the injectant and the oncoming supersonic or hypersonic stream predicted the observed experimental facts rather successfully: the viscous turbulent mixing model predicts the straight shock wave observed for uniform injection through a porous flat plate, and yields quantitatively correct results for the induced streamline deflection angles and surface pressures caused by mass injection.

However, the analysis of Part I was restricted to two-dimensional shapes. Thus the present study was undertaken—based on the same physical turbulent mixing model as was used for the two-dimensional analysis—in order to determine the effects of surface mass injection for the more practical case of axially symmetric bodies.

Most other theoretical studies[2–8] of the surface mass injection problem have as their basis, so far, an inviscid flow model in which the effects of laminar or turbulent mixing are relegated to a secondary role, being important only in a thin mixing layer surrounding the dividing stramline which separates the injected fluid from the external fluid. At high Mach numbers, however, turbulent dissipation can heat the gas considerably and the resulting increase in injectant volume can by itself cause significant induced streamline displacements and corresponding pressure increases, even for quite small injection rates.

One of the few studies which is not based on an inviscid model is that of Inger and Gaitatzes,[9] who incidently also give a rather comprehensive review and criticism of previously published studies based on inviscid models. Some of their results, which are based on a laminar mixing model, are equivalent to some we derive in Sec. 2. In that section, in order to bring out the basic features of the problem and to determine the relevant similarity parameters, we derive a rather general expression for the induced streamline deflection angles and then look at several interesting cases for injection through the surface of a cone. These results appear to be independent of whether the flow is laminar or turbulent; hence, the apparent equivalence to some of the laminar results of Inger and Gaitatzes. However, in both derivations it is implicitly assumed that the self-similar solutions of the differential equations governing the mixing layer do exist. The existence of such solutions must be demonstrated, however. Therefore, in Sec. 3, we look for self-similar solutions of the turbulent mixing layer equations and obtain ordinary differential equations similar to those obtained for the two-dimensional case but with an extra term which accounts for the effects of transverse curvature. The case of uniform blowing through the surface of a cone is included as a special case for which the entire flowfield is conical. Such a solution does not appear possible for a laminar mixing layer (e.g., see Ellinwood and Mirels[10–11]).

In Sec. 4, numerical results are obtained for the special case of uniform blowing through the surface of a cone, and these results show that the transverse curvature term has a significant effect on the properties of the turbulent shear layer, and in particular on the boundary-layer "blowoff" phenomena. Finally, in Sec. 5, the principal results of the present investigation are summarized.

‡ Unless otherwise indicated, a prime denotes differentiation with respect to η.

2. Flow Deflections and Surface Pressures Induced by a Uniform Surface Mass Injection Rate

It was found in our study of two-dimensional flows with surface mass injection[1] that a great deal of information could be obtained about the induced flow deflections and surface pressures without considering the details of the turbulent mixing process and without having to solve the governing differential equations. It turns out that the same is true for axisymmetric flows, although the problem is complicated somewhat by the transverse curvature effects.

We shall first derive an expression for the induced streamline deflection angle for a curved axisymmetric body with pressure gradient (Fig. 1), but then we shall specialize to consider in detail the simpler but important example of uniform mass injection over a semi-infinite cone. For that case the entire flowfield is conical, giving rise to a straight shock wave, a linearly growing mixing layer, and a uniform surface pressure.

One way to obtain the induced streamline deflection angle $\Theta(\delta)$ is by integrating the continuity equation from the body surface to the "edge" of the mixing layer (Figs. 1 and 2). One then obtains the following relation for $\Theta(\delta)$ in terms of a modified displacement thickness $\delta_a{}^*$ and the mass injection rate λ_e:

$$\tan\Theta(\delta) = \left(\frac{v}{u}\right)_{y=\delta} = \frac{d\delta_a{}^*}{dx} - \frac{(\delta - \delta_a{}^*)}{\rho_e u_e r_e}\frac{d\rho_e u_e r_e}{dx} + \frac{r_0}{r_e}\lambda_e \quad (1)$$

The modified displacement thickness $\delta_a{}^*$ is the same as that introduced by Fannelop[12] and is related to the perhaps more physically meaningful displacement thickness δ^* by the following expressions:

$$\delta_a{}^* = (r_0/r_e)(\delta_1 + \bar{\delta} - \delta)$$

where

$$\delta_1 = \delta^*[1 + (\delta^*/2r_0)\cos\alpha] \quad \text{and} \quad \bar{\delta} = \delta[1 + (\delta/2r_0)\cos\alpha]$$

Another useful relation which can be derived from the above expression is

$$\delta - \delta_a{}^* = (r_0/r_e)(\bar{\delta} - \delta_1)$$

A modified momentum thickness θ_a can be defined in a way analogous to the definition of $\delta_a{}^*$, and by integrating the momentum equation from the wall to $y = \delta$, we obtain the following equation for θ_a:

$$\frac{d\theta_a}{dx} = \frac{r_0}{r_e}\frac{c_f}{2} + \frac{r_0}{r_e}\lambda_e - \frac{\theta_a}{\rho_e r_e}\frac{d\rho_e r_e}{dx} - \left(\delta_a{}^* + 2\theta_a - \frac{\delta^2}{2r_e}\cos\alpha\right)\frac{1}{u_e}\frac{du_e}{dx} \quad (2)$$

A simple relationship can also be found between the modified momentum thickness θ_a and the momentum thickness θ:

$$\theta_a = \theta\{1 - [\delta - (\theta/2)] \cos\alpha/r_e\} = (r_0/r_e)\theta_1 \qquad (3)$$

where

$$\theta_1 \equiv \theta[1 + (\theta/2r_0) \cos\alpha]$$

By making use of Eq. (2), we can now write Eq. (1) for the induced streamline deflection angle Θ in terms of the modified form factor $H_a = \delta_a^*/\theta_a$:

$$\tan\Theta = H_a \frac{r_0}{r_e}\frac{c_f}{2} + (H_a + 1)\frac{r_0}{r_e}\lambda_e - \frac{\delta}{\rho_e r_e}\frac{d\rho_e r_e}{dx} + \frac{\delta_a^*}{H_a}\frac{dH_a}{dx} - \left[\delta + (H_a + 1)\delta_a^* - \frac{H_a \delta^2}{2r_e}\cos\alpha\right]\frac{1}{u_e}\frac{du_e}{dx} \qquad (4)$$

We now consider the case of uniform mass injection over the surface of a semi-infinite cone (Fig. 3). In that case r_0, r_e, and δ grow linearly with the distance x along the surface, and ρ_e, u_e, and $\alpha = \theta_c$ are constants. One then finds that

$$\tan\Theta = H_a(r_0/r_e)c_f/2 + (H_a + 1)(r_0/r_e)\lambda_e - (\delta/x) \qquad (5)$$

The form factor H_a can be related to the equivalent two-dimensional incompressible flow form factor H_i by employing the following modification of the transformation used for the two-dimensional case: $dY = (\rho_e a_e r/\rho_2 a_2 L)dy$, (Sec. 3), and then making use of the Crocco energy integral. We thus obtain

$$\theta_a = \int_0^\delta \frac{\rho u r}{\rho_e u_e r}\left(1 - \frac{u}{u_e}\right) dy = \frac{\rho_2 a_2 L}{\rho_e a_e r_e} \times \int_0^{\delta_i} \frac{U}{U_e}\left(1 - \frac{U}{U_e}\right) dY = \frac{\rho_2 a_2 L}{\rho_e a_e r_e}\theta_i \qquad (6a)$$

and

$$\delta_a^* = \int_0^\delta \left(1 - \frac{\rho u r}{\rho_e u_e r_e}\right) dy = \frac{\rho_2 a_2 L}{\rho_e a_e r_e}\int_0^{\delta_i} \times \left(\frac{T}{T_e} - \frac{U}{U_e}\right) dY + \frac{\delta^2}{2r_e}\cos\alpha \qquad (6b)$$

where $u/u_e = U/U_e$. According to the Crocco integral,

$$\frac{T}{T_e} = \frac{T_w}{T_e} + \left(1 - \frac{T_w}{T_e}\right)\frac{U}{U_e} + \frac{\gamma - 1}{2} M_e^2 \left[\frac{U}{U_e} - \left(\frac{U}{U_e}\right)^2\right] \qquad (7)$$

and using Eqs. (6) and (7), we obtain

$$\delta_a^* = \frac{\rho_2 a_2 L}{\rho_e a_e r_e}\left(\frac{T_w}{T_e}\delta_i^* + \frac{\gamma - 1}{2} M_e^2 \theta_i\right) + \frac{\delta^2 \cos\alpha}{2r_e} \qquad (8)$$

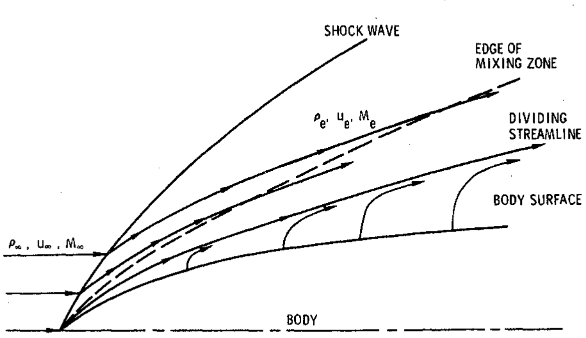

Fig. 1 Surface mass injection at supersonic speeds (schematic).

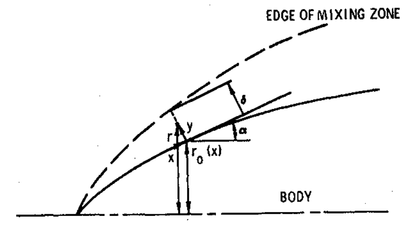

Fig. 2 Boundary-layer-type coordinate system for axially symmetric flow.

and

$$H_a^* = \frac{\delta_a^*}{\theta_a} = \frac{T_w}{T_e}H_i + \frac{\gamma - 1}{2}M_e^2 + \frac{\delta^2\cos\alpha}{2r_e\theta_a} = H_1 + \frac{\delta^2 \cos\alpha}{2r_e\theta_a} \qquad (9)$$

where

$$H_1 = \delta_1/\theta_1 = (T_w/T_e)H_i + [(\gamma - 1)/2]M_e^2$$

For a cone, we can solve for θ_a directly from the integrated momentum equation [Eq. (2)], which for conical flows becomes

$$d\theta_a/dx + (\theta_a/x) = r_0/r_e(c_f/2 + \lambda_e)$$

Thus for $\lambda_e \gg c_f/2$, we obtain§

$$\theta_a = (r_0/r_e)(\lambda_e/2)x = (r_0^2/r_e)(\mu_e/2 \sin\theta_c) \qquad (10)$$

Substituting Eq. (10) into Eq. (9), we obtain the following expression for H_a for the case of a cone:

$$H_a = \frac{T_w}{T_e}H_i + \frac{\gamma - 1}{2}M_e^2 + \left(\frac{\delta\cos\theta_c}{r_0}\right)^2\frac{\tan\theta_c}{\lambda_e} \qquad (11)$$

Thus for $\lambda_e \gg c_f/2$, Eq. (5) becomes

$$\tan\Theta(\delta)\left[1 + \frac{\delta\cos\theta_c}{r_0}\right] = \left[\frac{T_w}{T_e}H_i + \left(1 + \frac{\gamma - 1}{2}M_e^2\right)\right]\lambda_e - \left(\frac{\delta\cos\theta_c}{r_0}\right)\tan\theta_c \qquad (12)$$

The remaining task is to derive a relation between $\delta \cos\theta_c/r_0$, M_e, and equivalent two-dimensional incompressible flow quantities. To do so, we write

$$\delta = \frac{\rho_2 a_2 L}{\rho_e a_e r}\int_0^{\delta_i}\frac{\rho_e}{\rho}dY = \frac{\rho_2 a_2 L}{\rho_e a_e r_0}\int_0^{\delta_i}\frac{T}{T_e}dY$$

Solving for δ and using the Crocco integral [Eq. (7)], we obtain the following expression for $\delta \cos\alpha/r_0$:

$$1 + (\delta\cos\alpha/r_0) = [1 + (2\theta_a r_a/r_0^2)F\cos\alpha]^{1/2}$$

and after substituting in the expression for θ_a for a cone [Eq. 10)], we obtain

$$1 + (\delta\cos\theta_c/r_0) = (1 + \lambda_e \cdot F\cos\theta_c)^{1/2} \qquad (13)$$

where F and z are defined in the Nomenclature list.

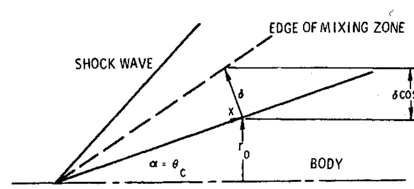

Fig. 3 Cone coordinate system.

§ For smaller blowing rates, we can include the effect of c_f approximately by replacing c_f by a constant mean value of c_f. In that case, $\theta_a = (r_0/r_e)(1 - B')(\lambda_e/2)x$, where $B' = -\frac{1}{2}c_f/\lambda_e$.

The induced streamline inclination $\Theta(\delta)$ can now be written entirely in terms of the edge Mach number M_e, the wall temperature ratio T_w/T_e, and the two-dimensional incompressible flow parameters H_i and z. Thus, using Eqs. (12) and (13), we obtain

$$\tan\Theta = \frac{\left[\frac{T_w}{T_e} H_i + \left(1 + \frac{\gamma - 1}{2} M_e^2\right)\right] \lambda_e - [(1 + \lambda_e F \cot\theta_c)^{1/2} - 1] \tan\theta_c}{(1 + \lambda_e F \cot \theta_c)^{1/2}} \tag{14}$$

or

$$\tan\Theta + \tan\theta_c = \tan\theta_c \left\{ \frac{1 + \lambda_e \cot\theta_c \left[\frac{T_w}{T_e} H_i + 1 \left(1 + \frac{\gamma - 1}{2} M_e^2\right)\right]}{\left\{1 + \lambda_e \cot\theta_c \left[\left(\frac{T_w}{T_e} + z\right) H_i + \frac{\gamma - 1}{2} M_e^2\right]\right\}^{1/2}} \right\}$$

We now consider the case of a slender cone in hypersonic flow in which case hypersonic-small-disturbance theory[14] is applicable for determining the "external" flow. Then $\tan\Theta(\delta) \cong \Theta(\delta)$, $\tan\theta_c \cong \theta_c$, $u_e \cong u_\infty$, and as in the two-dimensional case, the following expressions may be derived:

$$\lambda_e \left[\frac{T_w}{T_e} H_i + 1 + \frac{\gamma - 1}{2} M_e^2\right] \cong \lambda_\infty \frac{P}{P_e} \times$$

$$\left[\frac{T_w}{T_e} + \left(1 + \frac{\gamma - 1}{2} M_e^2\right)\right] \lambda_e F \cong \lambda_\infty \frac{P_\infty}{P_e} \times \left[\left(\frac{T_w}{T_\infty} + z \frac{T_e}{T_\infty}\right) H_i + \frac{\gamma - 1}{2} M_\infty^2\right]$$

We also write down the hypersonic-small-disturbance expressions for the pressure:

$$\frac{2}{\gamma}\left(\frac{P_e}{P_\infty} - 1\right) \cong \frac{4}{\gamma + 1} (K_s^2 - 1) + 2(K_s - K)^2 [(\gamma + 1)/(\gamma - 1)] \{1 + [2/(\gamma - 1)](1/K_s^2)\}$$

where

$$\frac{K_s}{K} = \frac{\gamma + 1}{\gamma + 3} + \left[\left(\frac{\gamma + 1}{\gamma + 3}\right)^2 + \frac{2}{\gamma + 3} \frac{1}{K^2}\right]^{1/2}$$

and

$$K = K_c + K_I = M_\infty\theta_c + M_\infty\Theta(\delta) \tag{15}$$

Therefore,

$$P_c/P_\infty = F(K_c + K_I)$$

With the above approximation, Eq. (14) may be written in the following form:

$$K_I + K_c = \frac{K_c \left[1 + \frac{M_\infty\lambda_e}{\frac{P_e}{P_\infty} K_c} \left\{\frac{T_w}{T_\infty} H_i + \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)\right\}\right]}{\left[\frac{M_\infty\lambda_e \left\{\left(\frac{T_w}{T_\infty} + z \frac{T_e}{T_\infty}\right) H_i + \frac{\gamma - 1}{2} M_\infty^2\right\}}{\frac{P_e}{P_\infty} K_c}\right]^{1/2}} \tag{16}$$

Equation (18) can be simplified even more if we introduce a cold-wall hypersonic approximation, namely

$$\frac{(T_w/T_\infty)H_i + 1}{[(\gamma - 1)/2]M_\infty^2} \ll 1 \quad \text{and} \quad \frac{[(T_w/T_\infty + z\, T_e/T_\infty)]H_i}{[(\gamma - 1)/2]M_\infty^2} \ll 1$$

In that case, Eq. (16) becomes

$$K_I + K_c = K_c \left[1 + \frac{\Lambda_\infty}{(P_c/P_\infty)K_c}\right]^{1/2} \tag{17}$$

or

$$(P_e/P_\infty)K_I(1 + \tfrac{1}{2}K_I/K_c) = \tfrac{1}{2}\Lambda_\infty$$

where $\Lambda_\infty = [(\gamma - 1)/2]M_\infty^3\lambda_\infty$ is the hypersonic blowing parameter found previously for the two-dimensional case.

We may now look at several interesting cases: first of all, let us consider the case where the induced streamline deflection angle due to the surface mass injection is "small" compared to the cone angle. In that event, $K_I/K_c \ll 1$. This also corresponds to the case where the mixing-layer thickness δ is much less than the cone radius r_0. From Eq. (17) we obtain the following expression:

$$K_I P_c/P_\infty \cong \Lambda_\infty/2 \qquad [(K_I/K_c) \ll 1] \tag{18}$$

The corresponding expression for the two-dimensional (wedge) case is

$$K_I P_c/P_\infty \cong \Lambda_\infty \qquad [(K_I/K_c) \ll 1]$$

As a second limiting case, let us consider the case where the induced streamline deflection angle is much larger than the cone angle. Then $K_I/K_c \gg 1$ and [Eq. (19)],

$$(K_I^2/K_c)(P_e/P_\infty) \cong \Lambda_\infty \qquad [(K_I/K_c) \gg 1] \tag{19}$$

There are now two interesting subcases to look at within this special case of $K_I/K_c \gg 1$. The first is the case where both the cone angle and induced streamline angle are very small but in a ratio such that the relation $K_I/K_c \gg 1$ still holds. Then $P_e/P_\infty \cong 1$ and Eq. (19) becomes

$$K_I \cong (K_c\Lambda_\infty)^{1/2} \qquad [K_I \ll 1;\ K_c \gg 1;\ K_I/K_c \gg 1] \tag{20}$$

Equation (20) thus implies that $\Lambda_\infty \gg K_c$ for this case. We can also rewrite Eq. (20) in terms of M_∞, λ_∞, and θ_c to obtain the following expression for $\Theta(\delta)$:

$$\Theta(\delta) \cong M_\infty \left(\frac{\gamma - 1}{2} \lambda_\infty\theta_c\right)^{1/2} \qquad \left[K_I \ll 1;\ K_c \ll 1;\ \frac{K_I}{K_c} \gg 1\right] \tag{21}$$

The second subcase is the case where $K_c = 0(1)$ and $K_I \gg 1$. Then, from Eq. (15), we obtain for $K \cong K_I \gg 1$:

$$P_e/P_\infty \cong [\gamma(\gamma + 1)(\gamma + 7)/(\gamma + 3)^2]K_I^2$$

Thus [Eq. (19)]

$$\left.\begin{aligned} K_I &\cong \left[\frac{(\gamma + 3)^2}{\gamma(\gamma + 1)(\gamma + 7)} K_c\Lambda_\infty\right]^{1/4} \\ \Theta(\delta) &\cong \left[\frac{(\gamma + 3)^2}{\gamma(\gamma + 1)(\gamma + 7)} \lambda_\infty\theta_c\right]^{1/4} \end{aligned}\right\} \left[K_c = 0(1);\ K_I \gg 1;\ \frac{K_I}{K_c} \gg 1\right] \tag{22}$$

The corresponding relation for the two-dimensional (wedge) case is

$$\Theta(\delta) \cong \{[(\gamma - 1)/\gamma(\gamma + 1)]\,\lambda_\infty\}^{1/3}$$

Thus for the axisymmetric case for $K_c = 0(1)$, $K_I \gg 1$, and $K_I/K_c \gg 1$, the induced angle $\Theta(\delta)$ has a one-fourth power law dependence on the blowing parameter as opposed to the one-third power law dependence for the two-dimensional case. In addition, $\Theta(\delta)$ has a weak dependence on the cone angle for the axisymmetric case, where the results for the two-dimensional case are independent of the wedge angle.

We also see from Eq. (22) that the assumption that $\Theta(\delta)$ be much larger than θ_c means that λ_∞ be much larger than θ_c.

3. Similar Turbulent Flows with Mass Injection

3.1 Transformation of Basic Equations of Motion and Boundary Conditions From Axisymmetric Compressible to Equivalent "Two-Dimensional Low-Speed" Flow

For the two-dimensional case, it was found that the compressible turbulent flow equations could be reduced to an equivalent "low-speed" form when the eddy diffusivity was of the following form:

$$\rho^2\epsilon = K_\theta \rho_e^2 u_e \theta G(Y/\theta_i)$$

where K_θ is a constant, and $G(Y/\theta_i)$ a suitably chosen function which allows for the influence of the surface in reducing the average turbulent eddy size near the wall.

For the axisymmetric case, we shall use the same form for the eddy diffusivity as for the two-dimensional case, except that we replace the two-dimensional momentum thickness θ by a momentum thickness appropriate to axially symmetric flows, namely θ_1. Thus we take

$$\rho^2\epsilon = K_\theta \rho_e^2 u_e \theta_1 G(Y/\theta_i) \tag{23}$$

We then apply the following transformations to the equations of motion to reduce them to equivalent low-speed, nearly two-dimensional forms:

$$\begin{aligned} \partial Y/\partial y &= (a_e/a_2)(\rho/\rho_2)r/L \\ dX/dx &= (a_e/a_2)(\rho_e/\rho_2)g(x) \end{aligned} \tag{24}$$

where $g(x)$ is to be determined later and would be different for laminar and turbulent flows.

The continuity equation is automatically satisfied by introducing a stream-function such that

$$u = (\rho_2 L/\rho r)(\partial\psi/\partial y) = (a_e/a_2)(\partial\psi/\partial Y) = (a_e/a_2)U \tag{24a}$$

$$v = -\frac{\rho_2 L}{\rho r}\frac{\partial\psi}{\partial x} = \frac{\rho_e}{\rho}\frac{L}{r}\frac{a_e}{a_2}g(x)V - U\frac{\partial Y}{\partial x} \tag{24b}$$

where

$$V = -\partial\psi/\partial X$$

The momentum thickness θ_1 is transformed as follows:

$$\theta_1 = (\rho_2 a_2 L/\rho_e a_e r_0)\theta_i \tag{25}$$

By utilizing the transformation [Eq. (24)] and introducing the expressions for u and v [Eqs. (24a) and (24b)] and turbulent eddy diffusivity [Eq. (23)], the momentum equation becomes

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = (1 + S)U_e\frac{dU_e}{dx} + \tilde{\epsilon}_0\frac{\partial}{\partial Y}\left[\frac{r_0}{Lg(x)}\frac{r^2}{r_0^2}G\left(\frac{Y}{\theta_i}\right)\frac{\partial U}{\partial Y}\right] \tag{26a}$$

where

$$\tilde{\epsilon}_0 = K_\theta U_e \theta_i \tag{26b}$$

and

$$S = h_s/h_{s_e} - 1 \tag{26c}$$

Similarly, if we assume equal momentum and thermal eddy diffusivities, the energy equation for thd total enthalpy takes the form

$$U\frac{\partial S}{\partial X} + V\frac{\partial S}{\partial Y} = \tilde{\epsilon}_0\frac{\partial}{\partial Y}\left[\frac{r_0}{Lg(x)}\frac{r^2}{r_0^2}G\left(\frac{Y}{\theta_i}\right)\frac{\partial S}{\partial Y}\right] \tag{27}$$

Thus we choose $g(x) = r_0/L$ and reduce the equations of motion to a nearly two-dimensional incompressible form.

In order to transform the quantity r^2/r_0^2 appearing in Eqs. (26a) and (27), we note that we may write it in the following form:

$$\frac{r^2}{r_0^2} = 1 - \left(1 - \frac{r^2}{r_0^2}\right) = 1 + \frac{2\cos\alpha}{r_0}\tilde{y} \tag{28}$$

where

$$\begin{aligned} \tilde{y} &= \int_0^y \frac{r}{r_0}\,dy \\ &= y\left(1 - \frac{y}{2r_0}\cos\alpha\right) = \frac{\rho_2 a_2 L}{\rho_e a_e r_0}\int_0^Y \frac{T}{T_e}\,dy \end{aligned} \tag{29}$$

Using the Crocco integral [Eq. (7)], we obtain

$$\tilde{y} = \frac{\rho_2 a_2 L}{\rho_e a_e r_0}\left\{\frac{T_w}{T_e}\int_0^Y\left(1 - \frac{U}{U_0}\right)dY + \frac{\gamma - 1}{2}M_e^2\int_0^Y\frac{U}{U_e}\left(1 - \frac{U}{U_e}\right)dy + \int_0^Y\frac{U}{U_e}\,dY\right\} \tag{30}$$

We may now use Eqs. (28) and (30) to obtain the following equations for r^2/r_0^2:

$$r^2/r_0^2 = 1 + \tilde{y}/\tilde{\sigma} \tag{31}$$

where

$$\tilde{y} = (\rho_e a_e r_0/\rho_2 a_2 L)\tilde{y} \tag{32a}$$

and

$$\tilde{\sigma} = \rho_e a_e r_0^2/2\rho_2 a_2 L\cos\alpha \tag{32b}$$

The momentum equation [Eq. (26a)] and the energy equation [Eq. (27)] now become the following:

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = (1 + S)U_e\frac{dU_e}{dX} + \tilde{\epsilon}_0\frac{\partial}{\partial Y}\left[G\left(\frac{Y}{\theta_i}\right)\frac{\partial U}{\partial Y}\right] + \frac{\tilde{\epsilon}_0}{\tilde{\sigma}}\frac{\partial}{\partial Y}\left[\tilde{y}G\left(\frac{Y}{\theta_i}\right)\frac{\partial U}{\partial Y}\right] \tag{33a}$$

$$U\frac{\partial S}{\partial X} + V\frac{\partial S}{\partial Y} = \tilde{\epsilon}_0\frac{\partial}{\partial Y}\left[G\left(\frac{Y}{\theta_i}\right)\frac{\partial S}{\partial Y}\right] + \frac{\tilde{\epsilon}_0}{\tilde{\sigma}}\frac{\partial}{\partial Y}\left[\tilde{y}G\left(\frac{Y}{\theta_i}\right)\frac{\partial S}{\partial Y}\right] \tag{33b}$$

The boundary conditions are obtained in the identical way as for the two-dimensional case,[1] thus only the final results will be given here. These are as follows:

$$\begin{aligned} \text{at } Y = 0:\quad & U/U_e = \tfrac{1}{2}c_f/\lambda_e \\ & S(X,0) = S_w(X) \text{ or } \partial S/\partial Y = 0 \end{aligned} \tag{34}$$

$$\text{at } Y \to \infty:\quad U = U_e(X), \qquad S = 0 \tag{35}$$

In addition,

$$\lambda_e = V(0)/U_e = (\lambda_e)_i \tag{36}$$

The "slip" boundary condition on U given by Eq. (34) takes into account the thin laminar sublayer, which is only important for "weak" blowing $[\lambda_e/c_{f_0} = 0(1)]$. An approximate method for estimating $c_f/c_{f_0} = F[\lambda_e(R_{e\delta})^{1/8}]$ is given in Ref. 1.

3.2 Similar Flows

As for the two-dimensional case, one looks for similar solutions of Eqs. (33a) and (33b) of the form

$$\psi(X,Y) = U_e\xi(X)f[Y/\xi(X)] = U_e\xi(X)f(\eta) \quad (37a)$$

$$S(X,Y) = S[Y/\xi(X)] = S(\eta) \quad (37b)$$

By introducing Eqs. (37a) and (37b) into Eq. (33a), one obtains the following equation for $f(\eta)$

$$\frac{\xi}{K_\theta U_e \eta_{\theta i}} \frac{dU_e}{dX} [(1 + S) - f'^2] + [G(\eta)f'']' + \left[\frac{1}{K_\theta U_e \eta_{\theta i}} \frac{d}{dX} (U_e\xi)\right] ff'' + \frac{\xi}{\tilde{\sigma}}\left[\frac{\tilde{y}}{\xi} G(\eta)f''\right]' = 0 \quad (38a)$$

where

$$\eta_{\theta i} = \frac{\theta_i}{\xi(X)} = \int_0^{\delta_i} f'(1 - f')d\eta \quad (38b)$$

Therefore, a requirement for similarity in addition to those for the two-dimensional case is

$$\tilde{\sigma}/\xi = \rho_e a_e r_0^2/2\rho_2 a_2 L\xi \cos\alpha = \text{const} = \hat{\sigma}$$

If the similarity requirements are satisfied, Eq. (38a) becomes

$$[G(\eta)f'']' + 1/\hat{\sigma}[\hat{y}G(\eta)f'']' + ff'' + \beta_i[(1 + S) - f'^2] = 0 \quad (39)$$

where [Eqs. (30) and (32a)],

$$\hat{y} = \frac{y}{\xi} = \frac{T_w}{T_e}\int_0^\eta (1 - f')d\eta + \frac{\gamma - 1}{2} M_e^2 \int_0^\eta f'(1 - f')d\eta + \int_0^\eta f'd\eta \quad (40)$$

The energy equation, Eq. (33b), becomes

$$[G(\eta)S']' + 1/\hat{\sigma}[\hat{y}G(\eta)S']' + fS' = 0 \quad (41)$$

and

$$\hat{\sigma} = \rho_e a_e r_0^2/2\rho_2 a_2 L(1 - \beta_i)K_\theta \eta_{\theta i} X \cos\alpha \quad (42a)$$

or [Eq. (26) with $g(x) = r_0/L$],

$$\frac{1}{\hat{\sigma}} = \frac{2\cos\alpha}{r_0}(1 - \beta_i)K_\theta\eta_{\theta i}\left[\frac{\rho_2 a_2 L}{\rho_e a_e r_0}\int_0^x \frac{\rho_e a_e r_0}{\rho_2 a_2 L} dX\right] \quad (42b)$$

The boundary conditions [Eqs. (34–35)] for these equations are exactly the same as for the two-dimensional case, i.e.,

$$f'(0) = -\tfrac{1}{2}\bar{c_f}/\lambda_e = (\bar{c_f}/c_{f_0})(\bar{c_{f_0}}/\lambda_e) \quad (43a)$$

$$V(0) = V_0 = -f(0)d/dX(U_e\xi) = K_\theta\eta_{\theta i}U_e f_0 \quad (43b)$$

or Eq. (36),

$$\lambda_e = K_\theta \eta_{\theta i} f_0 \quad (43c)$$

and

$$S(0) = S_w \text{ or } S'(0) = 0 \quad (43d)$$

$$\lim_{\eta\to\infty} f'(\eta) = 1; \quad \lim_{\eta\to\infty} S(\eta) = 0 \quad (43e)$$

Again, as in the two-dimensional case, these boundary conditions impose the following additional restrictions if similarity is to exist:

$$\lambda_e = \text{const and } S_w = h_w/h_{s_e} = \text{const}$$

The quantity $\eta_{\theta i}$ in Eq. (43c) is not an independent parameter, but is uniquely determined by the values of $f(0) = -f_0$ and $f'(0)$. This connection can be derived by integrating Eq. (39) across the turbulent layer. One obtains

$$(1 + \beta_i)\eta_{\theta i} = f_0[1 - f'(0)] - \beta_i\left(\eta_{\delta i}^* + \int_0^\infty S d\eta\right) \quad (44)$$

where

$$\eta_{\delta i}^* = \int_0^\infty (1 - f')dy$$

These relations, too, are identical to those derived for the two-dimensional case. When

$$\beta_i = 0, \ \eta_{\theta i} = f_0[1 + \tfrac{1}{2}(\bar{c_f}/c_{f_0})(c_{f_0}/\lambda_e)] \quad (45a)$$

and Eq. (43c),

$$f_0 = (\lambda_e/K_\theta)^{1/2}(1 + \tfrac{1}{2}(\bar{c_f}/c_{f_0})(\bar{c_{f_0}}/\lambda_e)^{-1/2} \quad (45b)$$

so

$$\eta_{\theta i} = (\lambda_e/K_\theta)^{1/2}[1 + \tfrac{1}{2}(\bar{c_f}/c_{f_0})(\bar{c_{f_0}}/\lambda_e)]^{1/2} \quad (45c)$$

4. Turbulent Flow over a Semi-Infinite Cone with Uniform Mass Injection

In Sec. 2 we considered various limiting cases for turbulent flow over a semi-infinite cone with uniform mass injection. All of these cases were for the hypersonic cold-wall slender cone, and under those basic assumptions the induced streamline deflection angles and relevant similarity parameters were derived for various limiting ratios of cone angle to induced deflection angle. The results obtained were independent of the form parameter H_i and of z.

For moderate Mach numbers and larger wall temperatures a more exact analysis is necessary; i.e., the full equation for determining $\Theta(\delta)$ [Eq. (14)] must be used. We must then determine H_i and z for various cone angles, freestream conditions, and cone temperatures. The necessary equations for doing this were set up in Sec. 3. In this section we shall specialize those equations to the case of a cone, and will present some results obtained from the numerical integration of the equations.

First of all, for a semi-infinite cone in supersonic flow, M_e is a constant and $\beta_i = 0$. Equation (39) thus reduces to the following:

$$[G(\eta)f'']' + 1/\hat{\sigma}[\hat{y}G(\eta)f'']' + ff'' = 0 \quad (46a)$$

The enthalpy equation, Eq. (41), is also of this form, which confirms the fact that the Crocco integral [Eq. (7)] holds.

Another equation is needed to solve for the two unknowns f and $\hat{y}$. That equation is Eq. (40). It is convenient for numerical purposes, however, to work with the derivative of that equation, namely,

$$\hat{y}' = T_w/T_e(1 - f') + [(\gamma - 1)/2]M_e^2[f'(1 - f')] + f' \quad (46b)$$

The boundary condition on $\hat{y}$ is $\hat{y}(0) = 0$. The boundary conditions on f are given by Eqs. (43a) and (43e).

Two more quantities must be known before Eqs. (46a) and (46b) can be integrated. These are the turbulent eddy diffusivity function $G(\eta)$ and the transverse curvature parameter $\hat{\sigma}$. Three different representative choices for $G(\eta)$ were investigated for the two-dimensional case in Ref. 1 and the justification for these choices was also given there. It turned out that two of the choices (case 1 and case 2) yielded values of the form factor H_i which were nearly the same for the two cases and which agreed well with available experimental data. The third choice, $G(\eta) = 1$, was the simplest but, not surprisingly, yielded the worst results. Since there is little or no experimental data available at the present time for determining the turbulent eddy diffusivity with mass injection for axially symmetric flows we shall, for the present, use the same form for $G(\eta)$ for the axially symmetric case as for the two-dimensional case. The case chosen for the present calculations is case 2 of Ref. 1, namely

$$G\left(\frac{Y}{\theta_i}\right) = 0.40\,\frac{Y}{\theta_i}\left(1 - 0.10\,\frac{Y}{\theta_i}\right) \text{ when } 0 \leqq \frac{Y}{\theta_i} \leqq 5$$

$$G\left(\frac{Y}{\theta_i}\right) = 1.0 \text{ when } \frac{Y}{\theta_i} \geq 5$$

Having determined a proper choice for $G(\eta)$, we now only need to determine the transverse curvature parameter $\hat{\sigma}$. That parameter, however, is given by Eq. (42b), which for constant edge conditions and for $r_0 = x \sin\theta_c$ (the case of a cone), yields

$$\hat{\sigma} = \tan\theta_c / K_\theta \eta_{\theta i} \tag{47}$$

where $\eta_{\theta i}$ is given by Eq. (45c) and f_0 by Eq. (45b).

The details of the numerical integration are the same as for the two-dimensional case and are described in more detail in Ref. 1.

Some of the results of the numerical integration of Eqs. (46a) and (46b) for $K_\theta = 0.06$ are presented in Figs. 4 and 5. Computations were made for edge Mach numbers of 5 and 10 and wall-to-edge temperature ratios of 1 and 5. Numerical solutions were obtained for values of the blowing parameter λ_e up to 0.25 except for the $M_e = 10$, $T_w/T_e = 5$ case. For that case it appears that no solution of Eqs. (46a) and (46b) exists which satisfies all of the boundary conditions at the wall and at the mixing layer edge if λ_e is greater than some value which lies between 0.20 and 0.25.

5. Summary and Conclusions

For axially symmetric as well as for two-dimensional bodies, significant streamline inclinations and surface pressures are generated by the action of viscous dissipation alone. For a "cold" cone in a hypersonic stream, simple relationships can be obtained between the induced streamline angle and the nondimensional blowing rate for certain limiting cases.

1) For $K_I/K_c \ll 1$:

$$K_I P_e / P_\infty \cong \Lambda_\infty / 2 \text{ for a cone}$$

whereas

$$K_I P_e / P_\infty \cong \Lambda_\infty \text{ for a wedge}$$

where Λ_∞ is the hypersonic similarity parameter for blowing found in Part I for the two-dimensional case:

$$\Lambda_\infty = [(\gamma - 1)/2] M_\infty^3 \lambda_\infty$$

and $K_I = M_\infty \Theta$; $K_c = M_\infty \theta_c$; P_e is the pressure on the surface, P_∞ the freestream static pressure, Θ the induced angle,

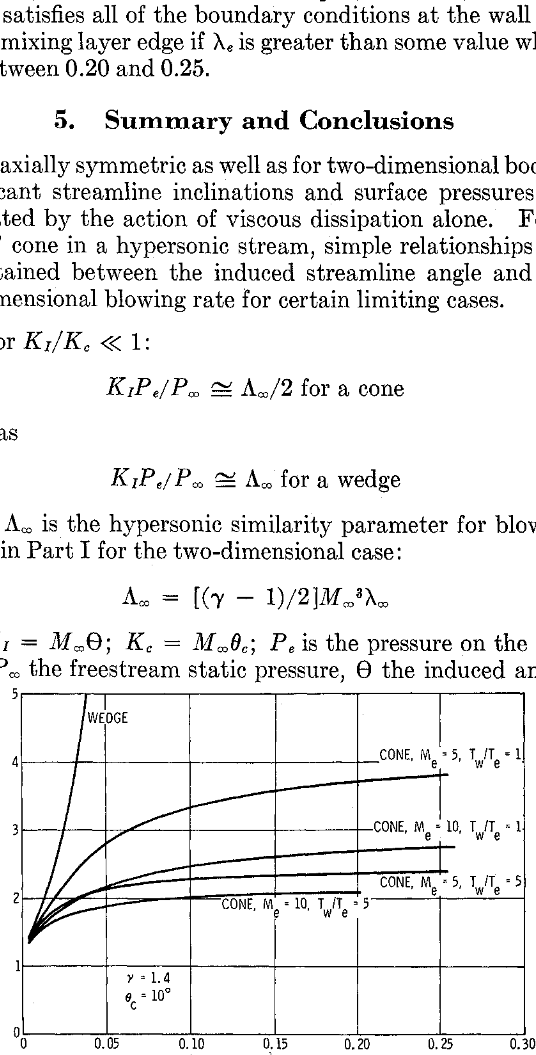

Fig. 4 Velocity profile shape factor $H_i = \delta_i^*/\theta_i$: surface mass injection at constant pressure.

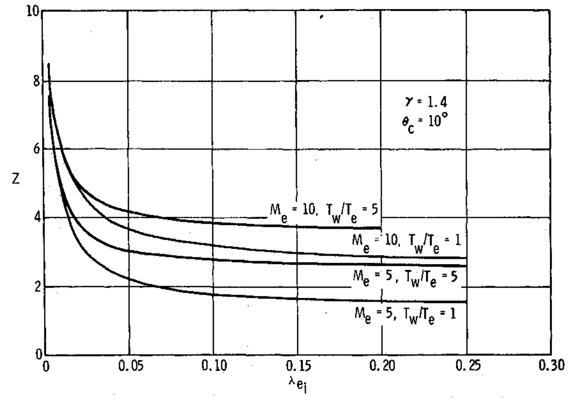

Fig. 5 $z = \frac{1}{\delta_i^*}\int_0^{\delta_i} \frac{U}{U_e}\,dY$: surface mass injection at constant pressure ($\delta_i = y$ at $U/U_e = 0.995$).

and θ_c the cone angle.

2) For $K_I \ll 1$, $K_c \ll 1$, but $K_I/K_c \gg 1$:

$$K_I \cong (K_c \Lambda_\infty)^{1/2} \text{ (implying } \Lambda_\infty \gg K_c)$$

$$\Theta \cong \{M_\infty[(\gamma - 1)/2]\lambda_\infty \theta_c\}^{1/2}$$

3) For $K_I \gg 1$, and $K_c = 0(1)$:

$$K_I \cong \{[(\gamma + 3)^2/\gamma(\gamma + 1)(\gamma + 7)] K_c \Lambda_\infty\}^{1/4}$$

or

$$\Theta \cong \{[(\gamma + 3)^2/\gamma(\gamma + 1)(\gamma + 7)]\Lambda_\infty \theta_c\}^{1/4}$$
$$= 0.910(\lambda_\infty \theta_c)^{1/4} \ (\gamma = 1.4) \text{ for a cone}$$

The corresponding expression for a wedge is

$$\theta = \{[(\gamma - 1)/\gamma(\gamma + 1)]\lambda_\infty\}^{1/3}$$
$$= 0.493\lambda_\infty^{1/3} \ (\gamma = 1.4) \text{ for a wedge}$$

When the surface is not "cold" relative to the freestream total temperature, the magnitude of the induced streamline deflection depends on the velocity profile shape factor H_i according to Eq. (14). It is interesting that for injection through the surface of a cone, unlike the case of injection through the surface of a wedge, the shape factor H_i appears to tend quickly toward a finite value as the blowing rate increases. For the wedge, H_i goes to infinity at a finite value of the blowing rate. Transverse curvature therefore seems to produce somewhat the same effects as does a favorable pressure gradient for two-dimensional flows.

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An Experimental Investigation of the Flowfield around a Yawed Cone

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An experimental investigation of the flowfield associated with a highly yawed five degree half-angle cone has been conducted in the wind tunnels at the U.S. Naval Ordnance Laboratory (NOL). The measurements, obtained for the most part at Mach 5, included surface pressure distributions, flow visualization photographs, and leeward side flowfield surveys. Analysis of these results indicates that the flowfield associated with a highly yawed cone at high supersonic velocities resembles that of a circular cylinder in a supersonic crossflow. The essential difference between these flowfields is the presence in the cone flowfield of a "vortical singularity like" gradient which separates the flow traversing the stronger portion of the shock wave on the windward side from the flow traversing the weaker portion of the shock wave on the leeward side.

Nomenclature

L = body length
M = Mach number
M_θ = $V/(\gamma P/\rho)^{1/2}$
P = pressure
P_{T_2} = Pitot pressure
R = distance measured perpendicular to the axis of the cone
R_b = local radius of the cone
Re = Reynolds number, $Re_{\infty},_X = \rho_\infty U_\infty X/\mu_\infty$
S = distance from vertex measured along a conical generator
T = temperature
U_∞ = uniform freestream velocity
V = elevation velocity component in a spherical coordinate system (positive in direction of increasing θ)
W = azimuthal velocity component in a spherical coordinate system
X = distance from tip of cone measured along the axis of symmetry of the cone
α = angle of attack
γ = ratio of specific heats
θ = elevation angle, measured from the axis of the cone, of a spherical coordinate system whose origin is at the apex of the cone
θ_c = cone half angle
μ = absolute viscosity
ρ = gas density
ω = azimuthal angle measured from the windward plane of symmetry

Subscripts

0 = stagnation conditions
∞ = undisturbed freestream conditions

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I. Introduction

The determination of the fluid dynamic properties of the flowfields around sharp right circular cones at supersonic and hypersonic velocities has been the subject of numerous experimental and theoretical investigations in the past.[1–13] Generally speaking, these previous investigations have demonstrated the adequacy of available techniques for calculating the inviscid flowfield and laminar boundary-layer properties on a yawed cone at small angles of attack (i.e., $\alpha \lesssim \theta_c$).[3–11] Furthermore, calculations of the inviscid flowfield and laminar boundary-layer properties on the windward side of a highly yawed cone (i.e., $\alpha \gtrsim 1.5\ \theta_c$) have also